

# THE COMMON POINTS IN RECENT THEORIES OF PROBLEM SOLVING

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Representing slightly differing points of view, at least three important experimental contributions have been made during the last decade to the knowledge of how man solves problems. It seems worth while to compare the results and methods to discover the points the resulting theories have in common and those in which they differ.

One of the first in order of time was Norman R. F. Maier's experiments. The simplest of his problems was to ask students to tie together two strings that hung from points of the ceiling so far apart that a man could not take hold of one and walk with it to the other without letting go of the first. The students were given a series of objects each of which ordinarily had some other function than that they were expected to put it to in the experiment. A record was taken of the attempts that they made and of their own observation of their own activity. Maier was impressed in his experiment by the fact that when an individual takes one attitude toward the problem, it is very difficult for him to change to another. His intention was that a pair of pliers which had been given with a miscellaneous set of objects should be tied to one of the strings; that it should be set swinging and that the worker should hold one string after starting the other swinging and catch the swinging string as it came towards him; then tie the two together. The pliers with the material that accompanied them usually suggested their use in catching or holding something and, in consequence, in relatively few cases were they regarded or thought of as material for making a pendulum. Even when suggestions were given

by swinging the string slightly, many of the workers did not think of the solution that was demanded. They could almost be told what to do without changing their attitude to regard the pliers as a pendulum bob.

Maier was impressed also by the fact that the memory of the subjects as to how the solution suggested itself was entirely unreliable. Even if they saw the solution only after a suggestion had been given, they would assert that they were not aided at all by the suggestion.

Maier calls the possibility of seeing the thing in a new way, taking a new direction. He insists that ability to change direction is an immediate capacity and that direction itself is a specific function. He furnished two bits of experimental evidence for the existence of direction. First he showed that ability to change direction is an important factor in problem-solving, possibly the most important. He demonstrated this by dividing the class into two groups; giving one half of the class the suggestion that whenever they were working for a solution and found themselves thinking in the same old tracks that they should try for new ones and that success was dependent upon ability to change the direction of thinking. The other half of the class were not given directions; they were left to their own devices. He found that the group that had been given the discussion emphasizing the importance of changing direction had a very much greater percentage (40% as against 20%) of individuals who succeeded in solving the problems than did the group that had not been given this hint. It is certainly very striking that so simple a suggestion should increase the successes in so high a degree.

The second came from a series of experiments on rats which gave evidence that direction in general depends upon experience. What depends upon experience is not the specific result but the tendency to be properly guided in direction. His method was to have a complicated path from table to table between the position in which a rat was placed and food. One group of rats was permitted to learn the path and another group was not. Then both groups had between 8% and 36% of the total brain cortex removed; the average amount removed was about 16%. After they were fully recovered they were

given a chance to find a similar route but with changes in the path. It was found that the rats that had learned before the operation could learn a new path with significantly fewer mistakes than those that had not learned. Since the path was new, there could be no survival of specific learning after the operation, but there was evidently some capacity that was developed by learning that persisted after the operation. Maier calls this something that is acquired by practice and retained, «direction». It is an ability to choose the right path more easily after than before the training.

In general Maier assigns to direction the most important function in problem solving. It controls the suggestion that shall present itself. Ability to change direction quickly decides whether an individual is a good or poor thinker. It is not said to be a faculty, but has some of the aspects of explanation that were once assigned to that term. He asserts, «The parts or experiences given together are not enough to bring about the solution of a problem. The part or experiences must be combined in a certain manner and a «direction» or way the problem is attacked seems to be a factor which determines the nature of the combination». Maier also found that none, or almost none, of his large group of subjects succeeded at the first attempt. All had a very large number of false trials before the correct solution presented itself. He nevertheless denies that trial and error is an explanation of problem solving.

Duncker also works from the general Gestalt point of view and experimentally. As opposed to Maier's, Duncker's experiments did not involve manipulation. In one group he asked his workers how they would solve a theoretical problem, in another he asked for the solution of problems in geometry or number theory. Typical of the first was a request to devise a method by which rays might be applied to a malignant growth in the stomach without injuring healthy tissue. Among the second group was to ask why a six-place number in which the first three and second three digits were identical (as 324324 or 713713) were divisible by 13.

Duncker found that the same form of approach was used in both kinds of problems. There was first a general notion of the field in which the solution must lie and of the direction

that should be taken to solve it. This general idea determines the details that shall be used in the solution. The first may be solved by preventing the rays from touching healthy tissue, or by protecting the healthy tissue by a covering, or, what was the solution that was accepted as correct, by permitting a number of rays to converge upon the malignant tissue from different directions so that the tumor alone will be affected by rays strong enough to destroy tissue. The thinkers who started with one general idea followed it through by presenting one series of special minor solutions in harmony with that. They would be little likely to think of an other solution or kind of solution until they had satisfied themselves that the first would not work.

In solving the problem as to why six-place numbers in which the second group of three is identical with the first group of three should be divisible by 13, there was a series of similar lines of approach. Some looked to see if each of the repeated three digits was divisible by 13, others looked to see if there were some criterion like the crossed sum rule that holds for divisibility by 9, others again sought for a common divisor of all numbers of the class that held itself be divisible by 13. Each of these general principles limited the field which must be thought through in detail in testing the different numbers. While the general principle was being tested it served as a purpose or question which controlled the associations that would arise and would also reject any suggestions that are not in harmony with that purpose. In this problem the third of these preliminary goals proved successful. In an experiment in which suggestions were given, the only one that aided an appreciable number of workers was that any six-place number in which the second three digits were identical with the first three would be divisible by 1001. This cue brought success to the majority.

Duncker then sees the problem solving process as one in which there are many tentative solutions, that are guided by a preliminary idea. He pictures the changing attitude that gives different lines of attack as changes (*Umstruktuerungen*) in the Gestalt, after the analogy of figure and ground. He does not pretend to know what it is that causes the Gestalt to undergo

a change, or what it is that makes the figure change to ground. He raised the question and ends with the statement that it depends upon the pregnancy of the suggestions, but pregnancy again depends upon many unanalyzable conditions. He falls into the common pattern of analyses of the thought in so far as he assumes that thinking starts with a definite problem that arises when the individual has a goal and does not know how the goal is to be attained. Thinking takes various directions in the attainment of its goal. The limits within which one will think are determined first by a preliminary analysis into immediate problems. These prepare the way for a restructuring of the problem. Various attempts are usually made to obtain the most satisfactory reorganization. Ultimately one develops that brings an acceptable solution. He leaves unexplained how the preliminary analysis arises and what it is that induces the restructuring.

Another important relatively recent experimental study of the problem solving process was made by Claparède. This was written in the light of the work of the other studies, although the work was begun much before them. Its title, 'The Genesis of the Hypothesis, indicates the author's belief that problem solving depends in the last analysis upon the formation and acceptance of an hypothesis. As material for study, Claparède asked his subjects to show how they could explain the significance of humorous drawings when they were shown only the first and last of a series or to supply a title to a single picture that was more or less ambiguous. Other tasks were to supply missing words in phrases or parts of drawings. The method of detecting the steps in reaching the conclusion was to ask the workers to speak aloud the different steps as if they were talking to themselves and to have a record made of the remarks. He calls it the method of spoken reflection and recommends it as an adequate means of becoming aware of the subsidiary thought processes.

Claparède's analysis of the action of problem solving is not so very different from that given by the preceding men except in the terms that are used. He finds that his observers begin with the problem more or less clearly dominant. Occasionally the process of observation or the first reading will bring the

answer in a flash. If it does not the worker infers, which apparently means additions are made to the immediate perception. When inference fails there begins a series of trials (*tatonnement*) in which numerous hypotheses are tried and rejected. *Tatonnement* is made different from trial and error principally in that it shows signs of being guided in a relatively restricted direction. Even here two kinds are recognized: Movements of thought that seem to be little guided or to be the results of pure chance (this is the usual trial and error) and those in which the successive attempts at solution are restricted to a relatively narrow field. Each attempt is itself an hypothesis that prepares the way for the accepted one and accepting an hypothesis constitutes the conclusion.

It should be said that the difference between the first sort of *tatonnement* that he calls unsystematic and the second called systematic is but a difference in degree. Even in the most systematic there is always an occasional idea that is far removed from the true course, that may even be absurd. It is also true that the most unsystematic shows some little sign of direction. The classification pretends to be no more than a rough grouping, but emphasized the presence of direction in the various attempts at solution.

Claparède makes no claim to explaining the factors that control the different hypotheses or the final selection of the true ones. He found some indications that the experiences of the preceding hours influence the character of the suggestions that are made. A student who had come from a sitting with Coué interpreted a picture of numerous men with long hair as waiting for an hypnotic session was given as one instance of the influence of experience. He asserts specifically that the formation of the hypothesis escapes consciousness («la formation de l'hypothèse échappe à la conscience»). All the antecedents, the problem, the direction to be followed, the awareness of the coming of the hypothesis and of its correctness is conscious, but we are never aware of the formulation of the hypothesis itself. He mentions many factors that do not aid in the formations of hypotheses but few things that do aid. He quotes Souriau that invention is never the work of logic, of method. He speaks of a spark that provides the last step in

the appearance of the right hypothesis, but admits immediately after that this is purely a metaphor, that we have no direct knowledge of anything of the kind.

On the whole, Claparède gives approximately the same description of problem solving as Maier and Duncker. He is only a little more frank in admitting ignorance rather than assigning a name to it. Problem solving always involves consciousness of a question and that it is not yet solved. The solution itself may come immediately on its presentation or when it is read. This comes so readily and quickly that there is no chance to observe the process by which it acts. When delay occurs the thinker is aware of successive suggestions presenting themselves. Each suggestion is called an hypothesis and the series of suggestions or the process of having successive suggestions *tatonnement*. These suggestions may seem to follow no law to be purely chance, but most often they are in harmony with a logical limitation to a relatively restricted range. The range becomes more and more closely restricted as time goes on.

No conclusion is reached as to exactly how this limitation is brought about. It is related to the general constellation in Müller's sense in so far as a group of ideas tends to restrict the range of ideas that are aroused. It is also similar in effect to the actualization of knowledge or completion of the complex to which Selz assigns directive value in seeking solutions. He accepts the Gestalt view that the interpretation always comes suddenly and entirely transforms the situations. He questions that the use of insight or structuration is more than descriptive and definitely denies the theory that previous experience has no bearing upon it. As was seen, his experiments indicate that the directly preceding experiments do determine in part what hypothesis shall suggest itself. Claparède mentions the three theories and then concludes that there exists a real anticipating scheme that must be regarded as giving direction to thought. A question in turn grows from a need. The need seems to loom largest among the conditions that are mentioned as controlling the limits within which hypotheses will arise.

It is obvious from the review of all of these theories that at present psychology can do no more than draw the limits

within which a solution of the nature of problem solving must be sought. On the broad outlines all who have experimented agree. In the first place there must be a problem that is fully conscious. Then the solution of the problem never comes at once. Discovering a solution is always a tentative process with many trials. Occasionally the answer may be a matter of immediate knowledge. In that case most authorities would not call it problem solving, but merely recall. It would be memory, not reasoning. The third part of the problem is what determines when the suggestion shall be accepted. I have insisted in earlier writings that this is almost if not quite as important as the development of the suggestion itself. If one could have a perfect criterion of adequacy of the solution when it appeared and an infinite number of suggestions a satisfactory solution would be bound to come in time by the mere laws of chance. One would merely wait for the right solution to appear. Since there is some control over the course of thought, the time is measurably shortened in actual practice.

The control of the suggestions and the mechanisms that excite or give rise to the suggestions offer most disagreement and the explanations advanced by different authors may be studied more closely. First we may consider the processes by which the suggestions are thought to be offered. The earliest to be mentioned was by simple associations. This is too familiar to need to be discussed in detail. It assumes that all the elements used in solving a problem are derived from past experience and that the revival is always of some earlier aspect or event that has been at some time seen in connection with an element of the present problem. The more recent advocates of association as the factors in the problem solving process insist that there is always a control of the revival by tasks or attitudes. This decides what shall be suggested and prevents the process from being purely mechanical. It is a question whether the authors who speak of revival through association are insistent upon this explanation or use it because of the traditional assumption that all mental successions are due to association and problem solving should be no exception.

One difference between the associational school and the members of the Gestalt school who have been extremely pro-



minent in recent work on problem solving is that association would make the materials applied to the solution come from earlier experience. The Gestalt school emphasizes more the reorganization of experience. As opposed to the associationists, the Gestalt school finds an analogy for the process in a transformation of a Gestalt or a new structuring of the masses present in the material presented by the problem. Maier's statement is that an object presented in connection with a problem suddenly changes its figure. An aspect that has been prominent in the first interpretation shifts and another phase dominates. In his own experiment the pair of pliers on the table with other tools suddenly ceases to be something to hold with or press with and becomes a weight that may be used as a pendulum. The process is like the shifting of an ambiguous perspective from one interpretation to another.

When one attempts a choice between the two alternatives, there is no reason why both should not be taken. There seem to be many instances in the solution of the daily problems in which the situation suddenly suggests an older somewhat similar case. The new problem is solved when reference is made to the solution employed in the older instance. Franklin's thinking of a kite as a means of making electric contact with a cloud would be an obvious instance of recall. One finds just as many instances in which a change in the interpretation put upon the problem is an essential to the solution. Seeing the paired three-digit identical number as  $Z \times 1001$  might pass as an instance of the sort. The advocate of each theory would undoubtedly attempt to reduce his opponent instances to correspond to his own theory. The advocate of recall might argue that one must remember that 1001 is divisible by 13 as well as seeing that it was a factor in the given 323323. The advocate of the changing aspect theory might insist that seeing the problem as necessitating making contact with the cloud was an essential antecedent to thinking of the kite. One may grant that each of these reductions has some force and be the more ready to assert that in some instances of problem solving recall seems more important and in others that changing the aspect is more prominent.

One may even admit that actual empirical citing of instances in which the solution came immediately from the addition of an

old experience produced the solution is not invariably possible. At the same time, it must be granted that evidence is equally difficult to adduce that shifting of interpretation always precedes the flashing of the new solution. The recall of an old element has the advantage as an explanation that we can conceivably assign an antecedent for the appearance of the old experience, while the change in figure comes without appreciable cause.

There is complete agreement of all workers that suggestions present themselves in great number whenever a problem is to be solved. Seldom is the first suggestion or the first shift in interpretation successful in giving the solution. Trial and error, and tatonnement have been applied to the process by the observers who regard the successive trials as essential.

The Gestalt group give no name as a rule but all mention numerous trials in description of the process. Why each suggestion should arise is answered differently by the members of each school. Association and change of Gestalt are the two most extreme. The use of the term trial and error has been interpreted to mean that the thoughts result from pure chance. Claparède assumes that when the different suggestions vary widely they are chance. But really chance itself can mean nothing more than that the man giving the explanation is ignorant of the cause of the suggestion. In most cases of association the cue is not noticed or the time at which the cue was associated with the suggestion is not remembered. Change in figure or change from figure to ground is never assigned any cause. It is no more than a description of the process from the outside. No one ever attempts to say why. The other Gestalt explanation sometimes mentioned, that a Gestalt always tends to change from an imperfect to a perfect figure, is also only a description. Since the school offers no measure of a perfect or imperfect Gestalt, there is no guide or indication as to when a change can be expected to take place or the direction it will take when it occurs.

For our purposes it may be assumed that during the solution of a problem numerous suggestions present themselves. Some can be traced to older experiences, some consist in changing the attitude towards some of the factors involved in the problem. It may be a new use for some of the tools provided for the

construction, if it is a physical problem. It may be merely a transformation of the ideas involved. No rule can be laid down even in a single case that will guide the suggestion or that will enable one to anticipate what the suggestion is likely to be. One can say that owing to factors as yet unknown suggestion after suggestion appears until one comes that is accepted as adequate. That is approved and tested by construction if that be possible.

More open to observation is the control that is exerted upon the suggestion that appear. Again all recognize this control. All agree also that it is effective only within limits. The Würzburg school spoke of it as the effect of the task set, Ach called it a determining tendency, Selz applied it to reasoning directly as the process of schematic anticipation. An idea of the range within which the solution would be likely to arise restricts the actual suggestions to that field. Claparède recognized the same limitation due in part to preliminary idea and in part to the elimination of possibilities by the success or failure of each of the early attempts. He calls it *le schéma anticipateur*. He recognizes it as identical with the question envisaged as a directive agent. A physiological analogue for the influence of the task or question has been found by assuming that a question partially excites an area of the cortex and when an association presents itself within this partially aroused field it produces an effect more certainly than would any stimulus that would arouse any other field. This explanation probably can be applied to some cases.

On the whole each of the terms used indicates merely that there are effects of the problem set and of the context, subjective and objective, at the time a stimulus is applied that keeps the partial solutions thought of within a limited range. As the context changes or the attitude of the individual shifts the kinds of suggestions that appear change. All of these notions assume that there is an agent that acts to push the action in one direction or another, but the mechanism of pushing is left indefinite.

Norman Maier has used a slightly different analogy or picture for the same kind of guidance. He asserts that the essential phase of reasoning is the «direction». In one place he

defines direction as «the way the problem is attacked». In his discussion he used the term to describe the course search for a solution will take and tends to endow it with real force. He recognizes the similarity in its effects to the Aufgabe or task and with Selz' constellation. He argues that direction is not the same thing because Selz assumes that problem solving is association and implies past experience. Maier argues that reasoning must be entirely new, and hence something which merely controls associations will not explain problem solving. He admits in other connections that there are similarities between attitude or anticipatory schema and direction. They certainly accomplish the same function. Maier also admits that it is difficult to say how «direction» acts. He correlates it with stress in a field, but admits that such an explanation can be not more than tentative. His important application.

On the whole we must then accept as agreed upon by all workers that in the tentative approach to solution, the individual is guided by a number of factors. His anticipation of the end that must be attained, that by the question and context. Then from moment to moment different partial purposes appear as the problem is changed by partial successes or by the failure of some attempted course to follow through to its end. This leads to a gradual narrowing of the field within which suggestions arise, but can never be said to determine completely the particular suggestion that shall follow. How the control acts can not be more definitely stated. Given an attitude or task or expectation the direction of thought takes a given line. To call the process «direction» as Maier does probably differs from this only in naming the effect rather than a postulated cause and would be but another aspect of the same common condition.

An important element in the total process that has largely escaped emphasis, although many men mention it, is the process of choosing between the suggestions. The first tentative acceptance of a suggestion is immediate. It may be designated by the familiar term «belief» of logic and popular thought. Belief depends upon definite conditions as does the appearance of the suggestion. Like those also the conditions are not always evident at first glance. It seems to be an immediate response to each suggestion that comes to the man himself or is offered

by another man. I have pointed out elsewhere that doubt is the opposite of belief since doubt comes when one first believes and then disbelieves. Or when alternatives are presented one doubts when first one is accepted and then the other.

The conditions of belief have been shown to be closely related to the pleasantness of the suggestion. Lund found a high correlation between judgments of the pleasantness of propositions and belief in them. It can also be shown by analysis that belief depends definitely upon the experience of the individual with the same or similar statements. If one accepted that point of view it would follow that belief was a reaction between the experiences that were active in the individual when a statement was presented and the suggested solution. Change a man's experience by presenting new facts and you change his beliefs. Or if you will present a subject in a new way by emphasizing a different aspect of it you may induce belief where previously there was doubt or rejection of the principle. We have space here only to point out the importance of the acceptance or rejection of suggestions and not to go fully into the conditions, which would require a long treatise on belief.

Acceptance or rejection of an hypothesis in Claparède's use of the term is then the immediate goal of problem solving. The first suggestion of a solution is however but the first stage of the longer process that Claparède and Wallas have called verification. The final verification may be either through actual trial which is the method of the experimental tests or by means of the proofs of logic, deductive or inductive. In the mechanical problems that were used by Maier and others, the physical attainment of the end constitutes verification. If the problem is theoretical in character, one can only muster major premises that may be accepted by the individuals who oppose the solutions and let the belief that attaches to them extend to the accepted solution. Verification rather than problem solving has been the aim of formal logic in all its forms.

This fact of acceptance of some one solution offered is then the final determinant of reasoning. Unless an individual has a great reliability in accepting a true solution he will be a poor problem solver. Belief stands guard over the solution of the

problem and prevents being led astray by false hypotheses. The ability to reach conclusions and to pass upon their validity might be functions of different individuals. One could imagine a partnership of one man who was very versatile in reaching conclusions, and of another who was very competent and careful in passing upon the validity of his suggestion that would be highly effective in problem solving. Both functions are important and might well be very distinct in their operation. We frequently judge a man to be fertile in suggestion but not too reliable in passing upon suggestions. Others are always sound but with little originality. Both qualities are necessary in a good thinker.

It would seem then that while there are differences in emphasis between the men who worked on problem solving, there is also fundamental agreement as to the main points. All agree that the problem dominates the course of thinking and working. In all problems difficult enough to be truly reasoning, numerous attempts are made before a solution offers itself that will be accepted. The suggestions are restricted within fields that depend upon the way the worker thinks of the problem and usually upon a preliminary general notion of what the final solution will be. This anticipatory notion has been called task or set, the preliminary idea by Duncker, the schematic anticipation of Selz and Claparède. All are variations of the same notion. Maier's «direction» involves the same process but stated from a slightly different point of view, in terms of the product rather than of the process or antecedents.

Two general groups of more specific explanation of the origin of the suggestions can be recognized in trial and error on the one hand and association or structuring of the Gestalt or figure and ground on the other. The trial and error theory on analysis is merely a statement that many movements are made in the original use of the term and in the transfer to thinking that many suggestions present themselves before one is accepted. Sometimes it is said that the ideas come by chance, but this can mean only that no one knows why they present themselves. The theory would be of value only if coupled with the addition that from the ideas offered by chance and produced by the transforming and combining of patterns some process

selects those that are appropriate. This then is similar to the theory of evolution by survival of the fit. The real determinant is the selection not the forces that develop the forms.

The two theories that attempt to explain the separate suggestion leave them with almost as little real explanation. As applied to thinking or problem solving, the association theory simply assumes that all revival is by association and so thinking must fall in that class. None of the theorizers or experimenters really point out specific cases in which the problem recalls definite experiences from the past for its solution. The followers of the Gestalt explanation assert the material presented in the problem change their aspects, are restructured when the problem is solved. They, too, never give any reason or cause for the change. It is almost as completely descriptive as the trial and error theory. Observation would indicate that some solutions seem to depend more upon addition of old experiences, others upon changing the attitude towards the problem.

Whatever brings the fruitful idea, the fact that it is recognized as fruitful is the one feature common to all problem solving and more emphasis should be put upon it.

### Bibliography

- CLAPARÈDE, ED.: *La genèse de l'hypothèse, Étude expérimentale, Arch. de Psychol.*, T. XXIV, n° 93-94 (juin-sept. 1933).
- DUNCKER, KARI: *Zur Psychologie des produktiven Denkens 1935*: Berlin, Verlag von Julius Springer, S. 135.
- MAIER, NORMAN R. F.: *Reasoning in Rats and Human Beings, The Psychological Review*, Sept. 1937, 44, 5, 365—378.
- > Reasoning in humans: I. On direction, *J. Comp. Psychol.*, 1930, 10, 115—143.
- > Reasoning in humans: II. The solution of a problem and its appearance in consciousness, *J. Comp. Psychol.*, 1931, 12, 181—194.
- SELZ, O.: *Zur Psychologie des produktiven Denkens u. des Irrtums. Bonn, F. Cohen, 1922, pp. 688.*
- PILLSBURY, W. B.: *Recent Naturalistic Theories of Reasoning, Scientia*, July 1924.
- > *The Psychology of Reasoning 1910. D. Appleton and Company, New York and London, p. 306.*